Inside Out

'Procedural knowledge focuses on the ability to recall numerical facts and procedures. Numerical reasoning focuses on the ability to apply those facts and procedures within a wide range of contexts. It is about 'making sense'. It requires active engagement from the learner to think mathematically, choosing what to do and how to do it.' *[Welsh Govt. Teacher Support Material - What is Numerical Reasoning?]*

Generally, schools are competent at teaching facts and procedures. There is a variety of commercial publications whose contents have been thoughtfully structured to allow practice of and progression through mathematical skills and knowledge. They are designed to visually appeal to learners and may even offer 'numerical reasoning problems' in 'meaningful' contexts. Unfortunately, many are not used as the publisher/authors intended - the details of the recommended preparatory work remaining unread in a long-lost Teacher's Manual.

The teaching and learning of skills and knowledge required for developing numerical reasoning may not be so linear and straightforward. Part of the difficulty lies in the historical compartmentalisation of content into separate subjects in order to make a 'broad and balanced' curriculum more manageable. The natural connections between generic inter-disciplinary skills have been artificially severed. Therefore, to achieve success in numerical reasoning, learners need opportunity to be aware of and exercise these generic skills (e.g. higher order reading, critical thinking etc.) in a mathematical context, in addition to using numerical reasoning skills in contexts outside the dedicated maths lesson.

However, effective assessment of learners' numerical reasoning capabilities cannot be achieved unless they have opportunity to share what is going on in their minds when they attempt to solve problems. Hence the title '*Inside Out*'. We need to know *how* they are 'making sense' of the information presented to them...what kind of 'sense'...or if they are making any 'sense' at all! Here are some thoughts...

The Language Barrier
This is the downfall of some learners, even those who have the most secure knowledge of procedural maths. It may seem obvious that those who struggle to decode text are not going to be able to understand what is being asked of them. However, it may also be the case that so-called 'fluent' readers have not developed the necessary higher-order skills required to understand the problem. **At the very least, a learner needs to be able to identify and state to themselves exactly what they are being asked to discover/ solve.****

Reflective Question: How often do learners have the opportunity to verbalise, share and discuss their understanding of a numerical reasoning problem (NRP) ...with each other. with another adult?
Do they have any strategies for helping themselves e.g. circling, highlighting or annotating relevant or confusing information. Are those strategies personally effective?

(**Beware of 'Smart-Alec' problems, whose unnatural and specific choice of language is designed to mislead the unwary reader. These reduce NRPs to something akin to Christmas cracker tricks and serve to undermine confidence rather than test mathematical ability.)

Organise and Analyse
Once a learner understands what they are being asked to discover, they need to employ organisational and analytical skills. This involves identifying and sorting the useful information from the irrelevant. Some learners may need to record this graphically or diagrammatically, or restate information in a different written format e.g. a list, sequence, etc.. They need to answer the question...'**From the information, what do I know and how does it make sense?'**

Reflective question: How often do learners have the opportunity to organise information in a way that is helpful to them personally? Would they be able to explain their organisation to a peer or adult?

Make connections
In many ways this is where the 'magic' of numerical reasoning happens. It is where learners make decisions about **how to bridge the gap between what they know and what they need to find out.** They consider options which frequently involve identifying relationships and patterns, and choosing from a toolkit of possible procedures.

In a 'magic' show, the secret is in the preparation. A trick that takes seconds to perform, may need hours of preparation. And so it goes for numerical reasoning. For example, one solution to a problem may require the learner to be able to identify multiples of a certain number. Preparation would not only involve creating multiples, learning the multiples (perhaps by rote) but also practice at identifying them in games or the wider world. Another solution could depend upon the learner understanding the relationship between factors and multiples. Depth of understanding is developed from encountering the concept in different contexts and having the opportunity to discuss and 'play' with the concepts.

This stage is probably the most difficult for practitioners to develop. It requires skilful questioning which encourages exploration but does not spoon-feed. Of course, different learners will require different amounts of guidance. However, learners need to be able to verbalise and justify their thinking if practitioners are to uncover how best to assist. If that doesn't happen, we will not know where the skills/knowledge gap is located.

Explore and Explain
If learners can decide upon a process they think will provide a solution, can they record it? Even better, can they record it using mathematical symbols and conventions? Again, this is a skill developed through familiarity and practice. Giving verbal explanations of a process is no different. Where learners have had ample opportunity to discuss their thinking; share their rationale for choices; and have also been encouraged to use specific mathematical language, then, responding to requests for explanations becomes natural and non-threatening. However, when learners only encounter such requests for explanations during a high stakes test situation, their performance may not truly reflect their mathematical ability due to levels of anxiety caused by an unfamiliar situation.

One way of preparing learners for formal testing or any rich multi-disciplinary task is *'backward' design.*

‘Backward’ Design

‘Forward’ design could be thought of as planning a task which enables learners to demonstrate the knowledge, skills and attitudes they have acquired over a period of learning. It gives practitioners useful formative feedback on the success of their practice as well as useful formative information on and for learners.

‘Backward’ design begins with the end-point. This could be a yearly statutory test or assessment task. The question is asked, **‘What knowledge and understanding along with which skills and attitudes will learners need in order to be successful in this task?’**
After considering the answers to this question, a practitioner would know which attitudes, skills and knowledge to focus on developing prior to the task and then design opportunities for the learners to encounter, explore and purposefully reflect upon them.

In terms of numerical reasoning, there could be specific knowledge content e.g. multiples of 5. Learners may be familiar with this content - they may be able to count in 5s when instructed or know their times-table by rote. However, they may not be familiar with using the knowledge in different contexts. A number of opportunities could be built in to a design as maths ‘warm-ups’ (How many toes at that table?) or ‘real needs’ (How many minutes until...?). These activities could be followed by allowing learners to discuss their methods and give clear explanations to their peers, turning their thinking ‘inside out’! . Merits of different procedures and approaches could be discussed and evaluated etc.

If an assessment contains unfamiliar context or language (the language barrier), then introducing it to learners would need to be included in a design but not necessarily in a mathematical context. Familiarity with an object could be crucial to learners’ engagement and success with a problem. Being asked to solve a problem about the capacity of ‘cafetieres’ could be undermined if the majority of learners have never encountered the object except through a grainy, 2-D PowerPoint graphic during the lesson intro.

Designs may include opportunities to practice ‘test skills’ e.g. working to specific time limits or annotating/highlighting questions. These could be done individually or collaboratively. Purposeful reflection on these activities allows learners to express what they found easy, difficult or interesting and subsequently identify steps they can take to improve their performance.

Teaching to the test
Whether you agree or disagree with formal testing as a valid measure of achievement/attainment, statutory testing remains a reality. In tests where a regurgitation of facts/formulae is required, ‘shallow or surface learning’ (cramming) can produce artificially high results. However, numerical reasoning tests cannot be ‘crammed’ for (unless the questions are known ahead of time...which is cheating!).
To be successful in numerical reasoning tests, ‘deep learning’ needs to occur and the attitudes, knowledge and skills required not only for test success but for lifelong learning need to become embedded. And, as they become embedded, the necessity to revisit/re-learn some generic skills will diminish, allowing learners and practitioners more time and energy to focus on learning and developing the mathematical skills in meaningful contexts.